3.1 and 3.2 Derivatives! 2014-10-03 day 28

3.1/20 After t hours something is $s(t)=3t^2+t$ mi away from the start.

a) avg velocity on $[1,3]$ $= \dfrac{s(3)-s(1)}{3-1}$

$$= \frac{[3(3)^2+3]-[3(1)^2+1]}{2} = \frac{26}{2} = 13 \text{ mph}$$

b) inst velocity at $t=1$ $= \lim\limits_{t\to 1} \dfrac{s(t)-s(1)}{t-1}$

$$= \lim_{t\to 1} \frac{[3t^2+t]-4}{t-1} = \lim_{t\to 1} \frac{(t-1)(3t+4)}{(t-1)}$$

$$\begin{array}{r} 3t+4 \\ t-1 \overline{)\,3t^2+t-4} \\ -(3t^2-3t) \\ \hline 4t-4 \end{array}$$

$$= \lim_{t\to 1} 3t+4 = 7 \text{ mph}$$

3.1 and 3.2 Derivatives!

2014-10-03 day 28

3.1/19 height $s(t)=5t^3$ $t \in [0,40]$

a) $s(40)=5(40)^3=5(10)^3(4)^3=320{,}000$ ft

b) $\underset{[0,40]}{\text{avg } v} = \dfrac{s(40)-s(0)}{40-0} = \dfrac{320{,}000-0}{40}$

$= 8000$ fps

c) avg. vel during first 135 ft.

$135=5t^3$

$27=t^3 \Rightarrow t=3$

$27=t^3$

$\sqrt[3]{27}=t$

$\shortparallel$

$(27)^{(1/3)}$

$\underset{[0,3]}{\text{avg } v} = \dfrac{135-0}{3-0} = 45$ fps

d) inst vel at $t=40$ $= \lim\limits_{t\to 40} \dfrac{s(t)-s(40)}{t-40} = \lim\limits_{t\to 40} \dfrac{5t^3-5(40)^3}{t-40}$ (*)

$= \lim\limits_{t\to 40} 5\left(\dfrac{t^3-40^3}{t-40}\right)$

$= \lim\limits_{t\to 40} 5\left(\dfrac{\cancel{t-40}}{\cancel{t-40}}\right)(t^2+40t+40^2)$

$= 5(40^2+40^2+40^2) = 15(40^2)$ fps

3.1 and 3.2 Derivatives!

2014-10-03 day 28

3.1/14) $f(x) = \frac{1}{\sqrt{x}}$; $x_0 = 4$

a) slope of graph = slope of the curvy line ⇒ [[It does not compute, Will Robinson]] (Space Family Robinson)

⇒ slope of the tangent line

⇒ instantaneous r.o.c

$$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x \to x_0} \frac{\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x_0}}}{x - x_0}$$

$$= \lim_{x \to x_0} \frac{\frac{\sqrt{x_0} - \sqrt{x}}{\sqrt{x}\sqrt{x_0}}}{(x - x_0)} = \lim_{x \to x_0} \frac{\sqrt{x_0} - \sqrt{x}}{\sqrt{x}\sqrt{x_0}(\sqrt{x} - \sqrt{x_0})(\sqrt{x} + \sqrt{x_0})}$$

factor

-1

$$\frac{\frac{a}{b}}{c} \left(\frac{\frac{1}{c}}{\frac{1}{c}}\right) = \frac{a}{bc}$$

$$= \lim_{x \to x_0} \frac{-1}{\sqrt{x}\sqrt{x_0}(\sqrt{x} + \sqrt{x_0})}$$

$$= \frac{-1}{\sqrt{x_0}\sqrt{x_0}(\sqrt{x_0} + \sqrt{x_0})} = \frac{-1}{2(\sqrt{x_0})^3}$$

$1 \quad 1 \quad (2\sqrt{x_0})$

1

$$\frac{-1}{2(\sqrt{x_0})^2\sqrt{x_0}} = \frac{-1}{2x_0\sqrt{x_0}}$$

3.1 and 3.2 Derivatives!

2014-10-03 day 28

3.1/14) $f(x) = \frac{1}{\sqrt{x}}$; $x_0 = 4$

a) slope of graph = slope of the curvy line ⇒ [[It does not compute, Will Robinson]]

Space Family Robinson

⇒ slope of the tangent line

⇒ instantaneous r.o.c

$$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x \to x_0} \frac{\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x_0}}}{x - x_0}$$

$$= \lim_{x \to x_0} \frac{\frac{\sqrt{x_0} - \sqrt{x}}{\sqrt{x}\sqrt{x_0}}}{(x - x_0)} = \lim_{x \to x_0} \left(\frac{\sqrt{x_0} - \sqrt{x}}{\sqrt{x}\sqrt{x_0}(x - x_0)}\right)\left(\frac{\sqrt{x_0} + \sqrt{x}}{\sqrt{x_0} + \sqrt{x}}\right)$$

conjugate

factor

$$\frac{\frac{a}{b}\left(\frac{1}{c}\right)}{c\left(\frac{1}{c}\right)} = \frac{a}{bc}$$

$$= \lim_{x \to x_0} \frac{\overset{-1}{(x_0 - x)}}{\sqrt{x}\sqrt{x_0}(\sqrt{x_0} + \sqrt{x})(x - x_0)}$$

$$= \lim_{x \to x_0} \frac{-1}{\sqrt{x}\sqrt{x_0}(\sqrt{x_0} + \sqrt{x})}$$

3.1 and 3.2 Derivatives!

2014-10-03 day 28

3.1/14) $f(x) = \frac{1}{\sqrt{x}}$; $x_0 = 4$

a) slope of graph = slope of the curvy line ⇒ [[It does not compute, Will Robinson]]

Space Family Robinson

⇒ slope of the tangent line

⇒ instantaneous r.o.c

$$\lim_{x\to x_0} \frac{f(x)-f(x_0)}{x-x_0} = \lim_{x\to x_0} \frac{\left(\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x_0}}\right)}{x-x_0} \frac{\left(\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{x_0}}\right)}{\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{x_0}}}$$

$$= \lim_{x\to x_0} \frac{\left[\frac{1}{x} - \frac{1}{x_0}\right]}{(x-x_0)\left(\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{x_0}}\right)} = \lim_{x\to x_0} \frac{\frac{x_0 - x}{x x_0}}{(x-x_0)\left(\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{x_0}}\right)}$$

$$= \lim_{x\to x_0} \frac{\overset{-1}{x_0 - x}}{x x_0 (x-x_0)\left(\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{x_0}}\right)}$$

$$= \lim_{x\to x_0} \frac{-1}{x x_0\left(\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{x_0}}\right)}$$

$$\frac{1}{\frac{1}{x}+\frac{1}{y}} \overset{?}{=} x+y$$

$$= \lim_{x\to x_0} \frac{-1}{x x_0 \left(\frac{\sqrt{x_0}+\sqrt{x}}{\sqrt{x}\sqrt{x_0}}\right)}$$

$$\frac{1}{\frac{1}{2}+\frac{1}{2}} = \cancel{2+2}$$

$$\frac{1}{\frac{a}{\frac{b}{c}}} = \lim_{x\to x_0} \frac{-\left(\frac{\sqrt{x}\sqrt{x_0}}{\sqrt{x_0}+\sqrt{x}}\right)}{4 x x_0} = \lim_{x\to x_0} \frac{-\sqrt{x}\sqrt{x_0}}{x x_0(\sqrt{x_0}+\sqrt{x})}$$

$$= \lim_{x\to x_0} \frac{-1}{\sqrt{x}\sqrt{x_0}(\sqrt{x_0}+\sqrt{x})}$$

$$\frac{\sqrt{x}}{x} = \frac{x^{1/2}}{x^1} = x^{\frac{1}{2}-1} = x^{-\frac{1}{2}} = \frac{1}{x^{1/2}} = \frac{1}{\sqrt{x}}$$

$$\frac{1}{2} \div \frac{2}{3} = \frac{1}{2}\cdot\frac{3}{2} = \frac{3}{4}$$

$$\frac{3}{6} \div \frac{4}{6} = \frac{\frac{3}{4}}{\frac{6}{6}} = \frac{\frac{3}{4}}{1} = \boxed{\frac{3}{4}}$$

3.1 and 3.2 Derivatives!

2014-10-03 day 28

3.1/12) $f(x)=x^2+3x+2$; $x_0=2$

a) $$\lim_{x\to a}\frac{x^2+3x+2-(a^2+3a+2)}{x-a}$$

$3x-3a$ $\quad$ $(x-a)(x+a)$

$$=\lim_{x\to a}\frac{(x^2-a^2)+3(x-a)}{x-a}=\lim_{x\to a}\left[\underbrace{\frac{x^2-a^2}{x-a}}_{x+a}+3\left(\frac{x-a}{x-a}\right)\right]$$

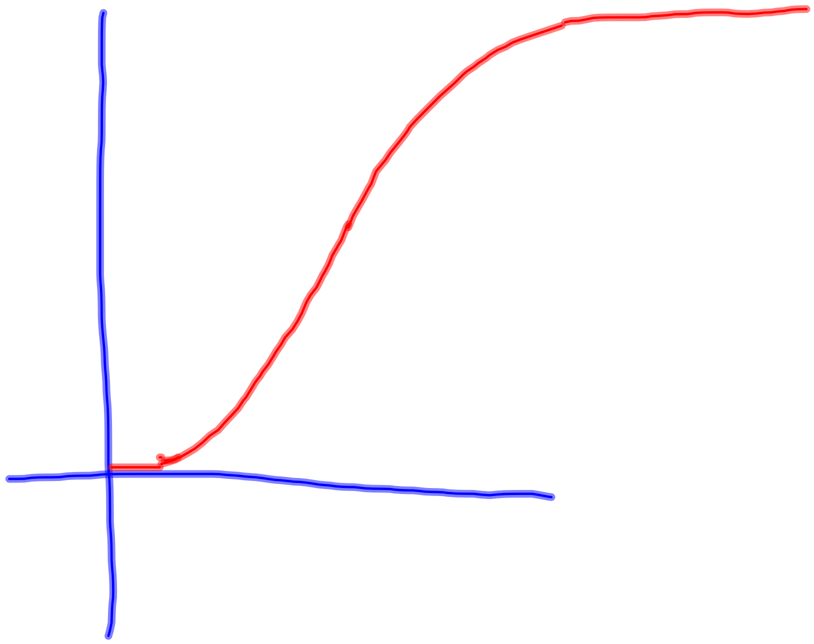
$$=\lim_{x\to a}\frac{(x-a)((x+a)+3)}{(x-a)}$$

$$=\lim_{x\to a}\left[(x+a)+3\right]=2a+3$$

3.1 and 3.2 Derivatives!

2014-10-03 day 28

3.1/4



3.1 and 3.2 Derivatives! 2014-10-03 day 28

3.2 A function that tells me the instantaneous rate of change at any point [[where it makes sense]] is called the derivative.

If the limit $\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = L$ exists [[i.e. = #]]

then that limit is called the derivative of $f(x)$ at $x = a$ and we write:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = L$$

limit No existe ⇒ No derivative